**Circles- Chords, Secants and Tangents**

**Circles and Chords**

- A ________________ is a segment that joins two points of the circle.
- A ________________ is a ____________ that contains the ________________ of the circle.
- A ________________ is a line that intersects a circle in ________ places and continues through the circle. A secant ________________ through a circle.

**Theorems:**

1. In a circle, a radius ______________________ to a chord __________________ the chord.
2. In a circle, a radius that __________________ a chord is ______________________ to the chord.
3. In a circle, the ______________ of a chord passes through the ______________ of the circle.

**Proof of Theorem 1:**

Given: \( \odot O, \overline{OD} \perp \overline{AB} \)
Prove: \( \overline{OD} \) bisects \( \overline{AB} \)

<table>
<thead>
<tr>
<th>Given</th>
<th>Prove</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \odot O, \overline{OD} \perp \overline{AB} )</td>
<td>( \overline{OD} ) bisects ( \overline{AB} )</td>
</tr>
<tr>
<td>Draw ( \overline{OA}, \overline{OB} )</td>
<td>Two points make a line</td>
</tr>
<tr>
<td>( \angle OEA, \angle OEB ) are right angles</td>
<td>( \perp ) lines form right angles</td>
</tr>
<tr>
<td>( \triangle OEA, \triangle OEB ) are right triangles</td>
<td>( \text{right angles contain one right angle} )</td>
</tr>
<tr>
<td>( \overline{OA} \cong \overline{OB} )</td>
<td></td>
</tr>
<tr>
<td>( \overline{OE} \cong \overline{OE} )</td>
<td></td>
</tr>
<tr>
<td>( \triangle OAE \cong \triangle OBE )</td>
<td></td>
</tr>
<tr>
<td>( \overline{AB} = \overline{BE} )</td>
<td></td>
</tr>
<tr>
<td>( E ) is the midpoint of ( \overline{AB} )</td>
<td>midpoint divides into ( m ) parts</td>
</tr>
<tr>
<td>( \overline{OD} ) bisects ( \overline{AB} )</td>
<td>bisector intersects at midpoint</td>
</tr>
</tbody>
</table>

**Theorem 4:**

In a circle, or congruent circles, __________________ chords are ___________________________ from the center.

**Problems using Theorem 4:**

Find \( x \).

**Theorem 5:**

In a circle, or congruent circles, congruent ________________ have congruent ________________.

**Theorem 6:**

In a circle, ______________________________ intercept congruent ______________. Note, the _______________ are not necessarily congruent, just the _______________ are.

**Problems using Theorems 5 and 6:**

Find the measure of each arc.

- \( x + 8 \) arc
- \( 3x \) arc

**Solution:**

\[ (x + 40)^\circ \]
Circles - Chords, Secants and Tangents

Closing: Complete the chart.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a radius ( \perp ) to a chord ( \perp ) the chord.</td>
</tr>
<tr>
<td>2</td>
<td>a radius that bisects the chord is ( \perp ) to the chord and they will therefore meet at ( \perp ) angles.</td>
</tr>
<tr>
<td>3</td>
<td>the ( \perp ) bisector of a chord can help you find the ( \perp ) of the circle.</td>
</tr>
<tr>
<td>4</td>
<td>If two chords are equidistant from the center of a circle, they are ( \perp ) congruent chords have ( \perp ) ( \perp ) between them are ( \perp )</td>
</tr>
<tr>
<td>5</td>
<td>( \perp ) bisector of a chord can help you find the ( \perp ) of the circle.</td>
</tr>
<tr>
<td>6</td>
<td>If two chords are parallel, the two ( \perp ) between them are ( \perp )</td>
</tr>
</tbody>
</table>

Homework Week 3: (Due 1/29/09) Complete Set A.

**Tangents and Circles**

A tangent to a circle is \( \perp \) in the plane of the circle that \( \perp \) the circle.

If you spin an object in a circular orbit and release it, it will travel on a path that is tangent to the circular orbit.

If a line is tangent to a circle, it is \( \perp \) to the \( \perp \) drawn to the point of tangency.

IF: \( AB \) is a tangent
\[ D \] is point of tangency
THEN: \( \overline{CD} \perp \overline{AB} \)

**Example 1:**

What must be the length of \( LM \) for this segment to be tangent line of the circle with center \( N \)?

Because the tangent is perpendicular (meets at a \( \perp \)) to the \( \perp \), we can use the \( \perp \) to find the length of \( LM \).

**Determining if a line is a tangent**

Because the tangent is perpendicular to the radius, we can use the Pythagorean Theorem to determine if a line is a tangent. If we do not get a true statement using the Pythagorean Theorem, the line is NOT a tangent.

How many, if any, of the circles above have tangent line? In both cases \( X \) is the center of the respective circles.

**Theorem:**

Tangent segments to a circle from the \( \perp \) are \( \perp \).

IF: \( AB \) is a tangent to circle \( O \) at \( A \)
\( \overline{CB} \) is a tangent to circle \( O \) at \( C \)
THEN: \( AB \equiv CB \)

(You may think of this as the "Hat" Theorem because the diagram looks like a circle wearing a pointed hat.)

\( \overline{OAB} \equiv \overline{OCB} \) by the \( \perp \) theorem.
**Circles - Chords, Secants and Tangents**

**Common Tangents:** Common tangents are lines or segments that are tangent to more than one circle at the same time.

<table>
<thead>
<tr>
<th>4 Common Tangents (2 completely separate circles)</th>
<th>3 Common Tangents (2 externally tangent circles)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 Common Tangents (2 overlapping circles)</th>
<th>1 Common Tangents (2 internally tangent circles)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**The only ways to have no Common Tangents:**

<table>
<thead>
<tr>
<th>0 Common Tangents (2 concentric circles)</th>
<th>0 Common Tangents (one circle floating inside the other, without touching)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Rules for Dealing with Chords, Secants, Tangents in Circles**

**Theorem 1:**
If two chords intersect in a circle, the _______________________ of the lengths of the _______________________ of one _________ equal the __________________ of the segments of the other.

**Intersecting Chords Rule:**
\[(\text{segment piece}) \cdot (\text{segment piece}) = (\text{segment piece}) \cdot (\text{segment piece})\]

**Example:**
In the circle below, the chord segments have the following lengths: A= 6, C=3, D=4. Use the theorem for the product of chord segments to find the value of B.

**Example:**
In the circle below, the chord segments have the following lengths: A= x + 4, B=3, D= 6. Use the theorem for the product of chord segments to find the value of C.

**Closing:** Complete the chart.

<table>
<thead>
<tr>
<th>Def.</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>tangents intersect a circle in _______ point.</td>
</tr>
<tr>
<td>2</td>
<td>a tangent is ______________________________ to the radius of the circle.</td>
</tr>
<tr>
<td>3</td>
<td>Tangent segments to a circle from the same external point are______________.</td>
</tr>
<tr>
<td>4</td>
<td>Because tangents are perpendicular to the radius, we can use the _______________________ to find missing lengths.</td>
</tr>
<tr>
<td>5</td>
<td>circles can have up to ______ common tangents if they do not intersect and are not inside one another.</td>
</tr>
<tr>
<td>6</td>
<td>Between two circles there can be ________________ tangents and ________________ tangents.</td>
</tr>
</tbody>
</table>

**Homework Week 3: (Due 1/29/09)**
Complete Set B.
**Example 1:**
Solving for x using the Secant-Secant Rule

Secant-Secant Rule:  
(whole secant)•(external part) = (whole secant)•(external part)

<table>
<thead>
<tr>
<th>whole secant</th>
<th>external part</th>
<th>whole secant</th>
<th>external part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 2:**
Solving for x using the Secant-Secant Rule

Secant-Secant Rule:  
(whole secant)•(external part) = (whole secant)•(external part)

<table>
<thead>
<tr>
<th>whole secant</th>
<th>external part</th>
<th>whole secant</th>
<th>external part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If a secant segment and tangent segment are drawn to a circle from the ________________, the product of the length of the ________________ segment and its external ____________ equals the ________________ of the length of the ________________ segment.

**Example:**

Secant-Tangent Rule:  
(whole secant)•(external part) = (tangent)$^2$

In the following problem, the red line is a tangent of the circle, what is its length?

<table>
<thead>
<tr>
<th>secant</th>
<th>external part</th>
<th>tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Theorem 3:**

Secant-Tangent Rule:  
(whole secant)•(external part) = (tangent)$^2$

If a secant segment and tangent segment are drawn to a circle from the ________________, the product of the length of the ________________ segment and its external ____________ equals the ________________ of the length of the ________________ segment.

**Example:**
You may have to solve tangent problems by factoring a quadratic equation.

- First, Outer, Inner, Last

Remember, the product of two binomials is a quadratic equation.

The middle term is formed by the __________ of the outer and inner products.

What is its length of the external part of the secant?

<table>
<thead>
<tr>
<th>whole secant</th>
<th>external part</th>
<th>tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Closing:** Complete the chart.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Theorem Diagram]</td>
<td>If two chords intersect in a circle, the ______ of the segments of one chord equal the ______ of the segments of the other.</td>
</tr>
<tr>
<td>![Theorem Diagram]</td>
<td>If two secant segments intersect the same __________ point, the product of one __________ segment and its external __________ equals the product of the other __________ segment and its external __________.</td>
</tr>
<tr>
<td>![Theorem Diagram]</td>
<td>If a secant and tangent segment are intersect the same __________ point, the product of the __________ segment and its external __________ equals the __________ of the tangent segment.</td>
</tr>
</tbody>
</table>

Homework Week 3: (Due 1/29/09)
Complete Set C.